Correlation between Interstellar Polarization and Dust Temperature: Alignment of Grains by Radiative Torques is Ubiquitous?

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Abstract

We investigate the efficiency of interstellar polarization p_λ/A_λ, where p_λ is the fractional linear polarization and A_λ is extinction, in 14 lines of sight as a function of wavelength λ. We have used the data of lines of sight to the Pleiades cluster obtained with the low-dispersion spectropolarimeter HBS as well as those in literature. It is found that the polarization efficiency p_λ/A_λ is proportional to $\exp(-\beta/\lambda)$ in wavelength $\lambda \approx 0.4 \sim 0.8\mu m$, where β is a parameter which varies from 0.5 to 1.2 μm. We find that β is negatively correlated with the dust temperature deduced from infrared data by Schlegel et al., suggesting that the polarization efficiency is higher in short wavelength for higher temperature. According to the alignment theory by radiative torques (RATs), if the radiation is stronger, RATs will make small grains align better, and the polarization efficiency will increase in short wavelength. Our finding of the correlation between β and the temperature is consistent with what is expected with the alignment mechanism by RATs.

Key words: ISM: dust, extinction — polarization — alignment mechanism

1. Introduction

Observed linear polarization in the light from distant stars, i.e., often called as "interstellar polarization", is interpreted as a phenomenon of dichroic extinction, and shows that interstellar grains are optically anisotropic and aligned, although the mechanism of the alignment has been still on debate (Lazarian 2007, for a recent review). The alignment of grains had been explained with the paramagnetic relaxation of thermally spinning grains that obtain angular momentum by collisions with gas particles (Davis & Greenstein 1951, hereafter DG). However, the DG mechanism is not efficient, and it cannot explain the interstellar polarization quantitatively. For more efficient alignment, Purcell (1979) assumed a spin-up of grain by the ejection of molecular hydrogen from grain surface (the "pinwheel mechanism"), though there still remain problems in quantitative explanations (Lazarian & Draine 1999; Roberge & Lazarian 1999).

Dolginov & Mitrofanov (1976) first pointed out that irregularly shaped grains that have "helicity" can spin up by radiative torques (hereafter RATs). More recently, Draine & Weingartner (1996) showed that RATs are very effective to align grains. Since magnetic moments within rotating grains are induced by the Barnett effect, the grains precess around the magnetic field, and the direction of alignment is usually parallel to the interstellar magnetic field (e.g. Draine & Weingartner 1997; Lazarian & Hoang 2007), i.e., the same direction as that by the DG mechanism. The alignment by RATs can be more efficient if grains have superparamagnetic inclusions (Lazarian & Hoang 2008) or if the pinwheel mechanism is working with RATs (Hoang & Lazarian 2009).

The efficiency of the RATs alignment varies with strength and spectral energy distribution of the radiation field, and thus the size of aligned grains should vary accordingly (Draine & Weingartner 1996; Cho & Lazarian 2007). Observationally, the maximum wavelength λ_{max} of polarization in dark clouds was shown to be correlated with extinction A_V in the V-band (Whittet et al. 2001; Andersson & Potter 2007). Andersson & Potter (2010) showed that grain alignment is enhanced by the stellar radiation in the vicinity of a young star HD 97300 in the Chamaeleon I cloud. For stars in the Taurus dark cloud, Whittet et al. (2008) showed that the polarization efficiency p_K/τ_K in the K-band, where τ_K is optical depth, decreases smoothly with A_V beyond the region where ice mantle feature was detected. This suggests that the alignment efficiency is not directly related to the state of grain surface, as is expected by the "pinwheel" alignment. Those observations suggest that the RATs alignment works in dark clouds and star forming regions. However, it is still not clear whether RATs alignment works or not in more diffuse clouds.

The Pleiades cluster is associated with the diffuse re-

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flection nebula, where grain alignment may be enhanced by strong stellar radiation if the alignment by RATs works. This motivated us to observe polarization in the lines of sight to stars in the Pleiades cluster with the low-dispersion spectropolarimeter HBS (Kawabata et al. 1999). In this Letter, using our polarimetric data and those available from literature, we investigate correlations between polarization quantities and dust temperature, because such correlations may be expected from the RATs alignment theory.

2.   Observations and Data Reduction

We observed 8 stars in the Pleiades cluster with the low-dispersion spectropolarimeter HBS (Kawabata et al. 1999) attached to the 1.88m telescope in Okayama Astrophysical Observatory, from October 30 to November 13 in 2008, and from January 13 to 19 in 2009. HBS has a super-achromatic half-wave plate and a quartz Wollaston prism, and it can measure linear polarization over a wide range of the optical region, from 0.4 to 0.8μm. We used a slit of 0.2mm width, yielding a wavelength resolution of $\sim 0.006\mu$m. After binning, we have obtained spectropolarimetric data with resolution of 0.02μm, and also synthesized V-band data. A unit of the observing sequence consists of successive integrations at $0°$, $22°.5$, $45°$, and $67°.5$ position angles of the half-wave plate. The exposure time per object was from 1800 to 17600 sec., depending on the brightness of object and also on weather conditions.

The instrumental polarization was evaluated with non-polarized standard stars, i.e., HD 432, HD 21447, HD 95418, HD 198149, and HD 210027 in 2008, and HD 21447, HD 20630, HD 95418, HD 114710 and HD 142373 in 2009 (Kawabata et al. 1999 and references therein). The standard deviations of the fractional Stokes parameters for those standard stars were 0.02-0.07% in the synthesized V-band. Since these values are larger than those expected from photon noise, i.e. 0.01-0.03% for each measurement, they can be interpreted as instrumental stability in polarization measurement. The photon noise for program stars was also small, ~ 0.01%. We thus consider that the accuracy in our polarimetry is mainly limited by the instrumental stability, and estimated the error for fractional polarization p_V to be 0.04%, which is a mean value of standard deviations for nonpolarized standard stars.

The position angle θ_V of linear polarization in the V-band was calibrated with polarized standard stars, i.e. HD 7927, HD 43384, and HD 204827 in both observational runs. Since HD 43384 shows small temporal variation in polarization (Hsu & Breger 1982; Matsumura et al. 1998), we considered it as a secondary standard, though no significant deviations were found from the tabulated value of $\theta_V = 170°$ in literature. We thus calibrated θ_V with the accuracy of $\sim 0.5°$.

From the observation of nonpolarized standard stars through a Glan-Taylor prism, we estimated the instrumental depolarization to be 0.95-0.99 depending on wavelength, and used it for calibration. The position angle θ_λ in wavelength $\lambda = 0.4 \sim 0.8\mu$m was calibrated with refer-
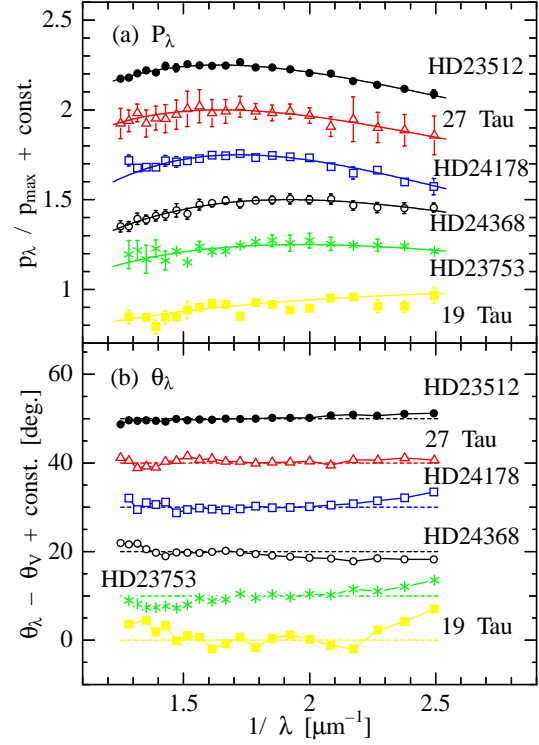
Fig. 1. Wavelength dependence of (a) fractional polarization p_λ and (b) position angle θ_λ for the Pleiades stars. Solid lines in (a) show results of fitting by equation (1), and dashed lines in (b) the values of θ_V. Data of each object are moved vertically in steps of 0.25 in (a) and $10°$ in (b).

ence to θ_V, with the same observation.

3.   Results and Discussion

3.1.   Fractional Polarization and Position Angle

The observed fractional polarization p_λ and position angle θ_λ are shown in Figure 1, except for HD 23985 and HD 24118 which show low polarization $\lesssim 0.1$%. We assume an empirical formula for p_λ by Serkowski et al. (1975):

$$p_\lambda = p_{max}\exp(-K\ln^2(\lambda/\lambda_{max})), \qquad (1)$$

where p_{max} is maximum polarization, K is a parameter that determines width of the curve, and λ_{max} is the wavelength at $p = p_{max}$. Those derived values of p_{max}, λ_{max}, and K are tabulated in Table 1. Figure 1a shows that p_λ is well expressed with equation (1). The position angle θ_λ is almost constant, though θ_λ of 19 Tau varies with λ in short wavelength $1/\lambda \gtrsim 2.3\mu$m^{-1} (Figure 1b).

Since the scattered light is often strongly polarized, it may affect the polarimetric results of nebulous objects, if it is not subtracted properly, and/or if the objects are surrounded by optically thick cloud, e.g. young stellar objects such as R Mon (Matsumura et al. 1999). However, the brightness of nebulosity around the stars in the Pleiades is not intensive compared with the stellar

Table 1. Polarization of Stars in the Pleiades Cluster

Name	$p_V{}^*$	$p_{max}{}^*$	$\lambda_{max}{}^\dagger$	$K^\dagger$	$\theta_V{}^\dagger$	Method	R_V	A_V	α	β	T_{dust}
(Sp.Type)	[%]	[%]	[μm]		[deg]			[mag]	[%/mag]	[μm]	[K]
19 Tau	0.26	0.28	0.36	0.26	140.2	1	2.74±.57	0.13	2.1±0.7	0.72±.01	20.1
(B5IV)	-	-	±.11	±.18	±4.4	2	3.20±.17	0.13	2.1±0.7	0.64±.01	-
27 Tau	0.34	0.35	0.61	0.91	112.6	2	3.20±.17	0.09‡	4.1±0.8	0.94±.08	19.7
(B8III)	-	-	±.03	±.45	±3.4	-	-	-	-	-	-
HD23512	2.34	2.38	0.61	1.06	27.2	1	3.48±.11	1.15	2.09±.07	0.82±.02	20.1
(A0V)	-	-	±.00	±.07	±0.5	2	3.20±.17	1.15	2.12±.07	0.94±.06	-
HD23753	0.27	0.27	0.51	0.57	104.8	1	3.17±.76	0.09	3.1±1.6	0.65±.14	19.3
(B8V)	-	-	±.01	±.16	±4.2	2	3.20±.17	0.09	3.1±1.6	0.67±.04	-
HD24178	0.50	0.50	0.58	1.38	128.2	1	2.57±.17	0.39	1.33±.13	1.00±.05	17.5
(A0)	-	-	±.00	±.13	±2.3	2	3.20±.17	0.39	1.33±.13	0.88±.06	-
HD24368	0.61	0.61	0.52	0.92	95.1	1	5.16±1.43	0.28	2.22±.31	$0.45\,^{<-.03}_{+.15}$	17.5
(A2V)	-	-	±.01	±.15	±1.9	2	3.20±.17	0.28	2.28±.32	0.74±.05	-

* Errors of p_V and p_{max} are estimated to be 0.04%. See text for details.

† Errors of λ_{max}, K, and θ_V are written in the 2nd line for each object.

‡ Deduced from the color excess by Crawford & Perry (1976), and its error is 0.02 mag. Other values of A_V are derived from 2MASS data, having errors of 0.04 mag.

light, typically ~ 20 mag/arcsec^2 in the B or V-band. We thus expect that the effect of the nebulosity is subtracted with the sky background in the reduction process. In addition, the spectral types of the stars are normal. Thus the observed polarization is expected to be mainly foreground interstellar in origin. Nevertheless, 19 Tau may have a component of non-interstellar origin, because the position angle is variable: θ_V was 114° in Markkanen (1977) and in Breger (1986), but it is $140\pm4°$ in our observation (Table 1). We thus exclude 19 Tau in the following discussion.

3.2. Polarization Efficiency

The A_V-dependence of λ_{max} in dark clouds suggests that the alignment of grains is induced by RATs (see Section 1). However, Andersson & Potter (2007) noticed that the extrapolated value of λ_{max} at $A_V = 0$ in each cloud is correlated with the mean value of the ratio of total to selective extinction R_V, where $R_V \equiv A_V/E_{B-V}$ and E_{B-V} is the color excess for $B-V$. Since R_V characterizes the extinction, this correlation means that λ_{max} depends not only on the alignment, but also on the size of total, i.e. aligned and nonaligned, grains. We thus use another quantity less affected by the variation of grain size.

Compared with λ_{max}, the polarization efficiency p_λ/A_λ should be less dependent on the variation of grain size, because such variation will be canceled in p_λ/A_λ. We thus explore the observed properties of p_λ/A_λ, expecting to obtain information on alignment. It should be noted, however, that Voshchinnikov & Das (2008) and Das et al. (2010) showed that p_λ/A_λ depends on grain size, shape, material, and other parameters. It would be possible to examine the properties of p_λ/A_λ in more detail with using light scattering calculations (e.g. Matsumura & Seki 1991, 1996; Matsumura & Bastien 2009), but it is beyond the scope of this Letter.

To evaluate extinction A_λ, we have used two methods:
Method 1: On the assumption that the λ-dependence

of A_λ is determined by R_V and scaled by A_V (Cardelli et al. 1989), we evaluate A_λ with interpolating the data of A_λ/E_{B-V} for $R_V = 2.1 \sim 5.5$ tabulated in Fitzpatrick (2004). The values of A_V and R_V are calculated by the formulae $A_V = 1.1E_{V-K}$ and $R_V = 1.1E_{V-K}/E_{B-V}$ (Whittet & van Breda 1980), respectively, where E_{V-K} is the color excess for $V - K$. We use B and V magnitudes in the Simbad database, while for the K band, we transform K_S magnitude in the Two Micron All Sky Survey (2MASS) into the K magnitude in the system of Koornneef (1983) with a formula by Carpenter (2001). We refer to Fitzgerald (1970) and Koornneef (1983) for the intrinsic colors of $B - V$ and $V - K$, respectively.

Since the errors of R_V derived with Method 1 are large for some stars (Table 1), we also use another method as below (Method 2). The error of K_S for 27 Tau was particularly large, ~ 0.3 mag., we could not obtain reliable results, and excluded 27 Tau in the following discussion.

Method 2: We assume that the extinction properties are not variable within the Pleiades cluster, and apply the extinction curve for HD 23512 to other stars, scaling it by the value of A_V deduced with Method 1. The extinction curve for HD 23152 is reduced by Fitzpatrick & Massa (2007), and most reliable among the Pleiades stars.

We explore not only the Pleiades stars, but also the stars for which polarization and extinction data are available from literature. We have used the polarimetric data of various stars by Weitenbeck (1999, 2004). The data of HD 29647 (Whittet et al. 2001) and HD 38087 (Serkowski et al. 1975) are used in addition to those from Weitenbeck (1999). Extinction data for those stars are cited from Fitzpatrick & Massa (2007). Also used are the data for high latitude clouds MBM 30 and MBM 20 (LDN 1642) by Seki & Matsumura (1996).

Figure 2 shows the λ-dependence of p_λ/A_λ, which is derived with Method 1. The values of $\log(p_\lambda/A_\lambda)$ decrease linearly with inverse wavelength $1/\lambda$, though slight devi-

Table 2. Polarization Properties deduced from Literature.

HD or SAO (Sp.Type)	α [%/mag]	β [μm]	$T_{\rm dust}$ [K]	Ref.[*]
14889 (K0)[†]	2.12±.21	0.46±.05	18.0	(1)
29647 (B8III)	0.65±.02	1.06±.05	15.0	(5)
—	0.62±.03	1.25±.08	—	(3)
30675 (B3V)	2.70±.13	0.83±.07	16.0	(5)
37367 (B2IV-V)	0.74±.04	0.76±.04	15.8	(4)
38087 (B5V)	1.47±.07	0.40±.06	21.0	(3)
—	1.51±.07	0.51±.03	—	(2)
192001 (O9.5IV)	0.84±.04	0.92±.05	19.8	(4)
193322 (O9V)	1.57±.10	0.91±.08	20.4	(4)
210121 (B3V)	1.76±.16	0.79±.11	17.4	(4)
216532 (O8.5V)	0.87±.02	0.74±.04	18.9	(4)
S149760 (K5)[†]	2.06±.21	1.03±.13	16.4	(1)

[*] References: (1): Seki & Matsumura (1996), (2): Serkowski et al. (1975), (3): Weitenbeck (1999), (4): Weitenbeck (2004), (5): Whittet et al. (2001). The data in the R-band of HD 38087 in Serkowski et al. (1975) is not used for fitting.
[†] Luminosity class is assumed as V, and the values of $(R_{\rm V}, A_{\rm V})$ are estimated to be (3.08±.20, 1.17±.11) for HD 14889, and (2.74±.54, 1.15±.12) for SAO 149760.
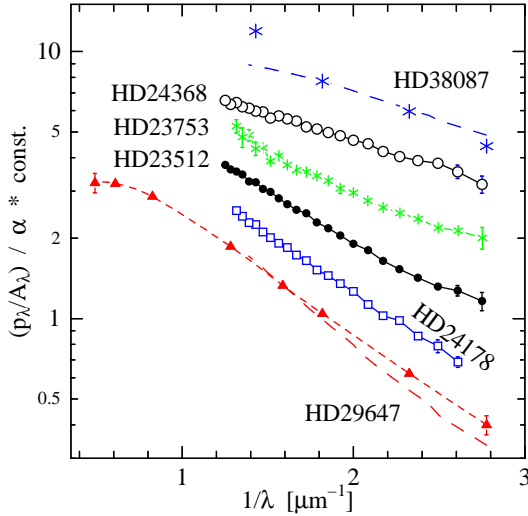
Fig. 2. Wavelength dependence of polarization efficiency p_λ/A_λ deduced with Method 1 (See text). Asterisks show the results of HD 38087 (Serkowski et al. 1975) and filled triangles HD 29647 (Whittet et al. 2001), while other symbols are for the Pleiades stars as in Figure 1. Long dashed lines are the results by Weitenbeck (1999). The data are normalized by α (see equation (2)), and moved vertically in steps of factor 1.5.

ations from the linear relation are found. We thus make linear fitting in $\lambda = 0.4 \sim 0.8\mu$m with the equation:

$$\ln(p_\lambda/A_\lambda) = \ln\alpha - \beta(1/\lambda - 1/0.55\mu\text{m}), \quad (2)$$

where λ is in μm, and α and β are parameters and tabulated in Tables 1 and 2.

3.3. Polarization Efficiency and Dust Temperature

To discuss the correlations between the polarization properties and dust temperature, we use the temperature $T_{\rm dust}$ by Schlegel et al. (1998). They deduced $T_{\rm dust}$ from COBE and IRAS data, on the assumption of λ^{-2} emissivity of grains in the infrared. Their data is homogeneous all over the sky, and thus suitable for our study that contains not only the lines of sight to Pleiades stars, but also those to other objects.

Between $\lambda_{\rm max}$ and $T_{\rm dust}$, we find a weak correlation in Figure 3a, and the correlation coefficient r is -0.30. The correlations between β and $T_{\rm dust}$ are much better, i.e., $r = -0.54$ (Method 1, Figure 3b) and $r = -0.57$ (Method 2, Figure 3c). It is remarkable that the relative positions of HD 210121 and HD 38087 are different between in Figure 3a and in Figure 3bc. This is caused by the different values of $R_{\rm V}$, i.e., $R_{\rm V}$ of HD 210121 is smaller (= 2.0, Fitzpatrick & Massa 2007), and that of HD 38087 is larger (= 5.8, Fitzpatrick & Massa 2007) than other objects ($R_{\rm V} \sim 3$).

We finally discuss the above mentioned correlations on the basis of the RATs alignment theory. Using their equation (5) in Cho & Lazarian (2007) and equation (67) in Draine & Weingartner (1996), with typical values for physical quantities in interstellar space tabulated in Table 2 of Lazarian & Hoang (2007), we can express the smallest size $a_{\rm lower}$ of aligned grains as

$$(a_{\rm lower}/1\mu\text{m}) = 0.089 \times (T_{\rm dust}/18\text{K})^{-2}, \quad (3)$$

where we assume that the efficiency Q_Γ for RATs is 0.1, and that the emissivity of grains is $\propto \lambda^{-2}$.

On the size distribution of aligned grains, Mathis (1986) showed that the observed polarization p_λ can be reproduced if the fraction $(1 - \exp(-(a/a')^3))$ of grains with radius a are aligned, where a' is a parameter for typical size of smallest aligned grains. Mathis (1986) then obtained the equation:

$$(a'/1\mu\text{m}) = 0.327 \times (\lambda_{\rm max}/1\mu\text{m})^{2.17}. \quad (4)$$

If we assume $a' = a_{\rm lower}$ (or $a' = 2a_{\rm lower}$), we can relate $\lambda_{\rm max}$ and $T_{\rm dust}$ with equations (3) and (4), and draw the short dashed (or long dashed) line in Figure 3a.

For the relation between β and $\lambda_{\rm max}$, we obtain

$$(\beta/1\mu\text{m}) = 1.79 \times (\lambda_{\rm max}/1\mu\text{m})^{1.39}, \quad (5)$$

with using equation (1) and extinction curve for $R_{\rm V} = 3.1$ (Fitzpatrick 2004). We then write β as a function of $T_{\rm dust}$, and draw short and long dashed lines, for $a' = a_{\rm lower}$ and $a' = 2a_{\rm lower}$, respectively, in Figures 3b and 3c.

Those lines in Figure 3 seem to follow the observations well, i.e., the correlation between $\lambda_{\rm max}$ and $T_{\rm dust}$, and that between β and $T_{\rm dust}$ can be explained by the RATs alignment theory. Since those data contain regions of various temperature, i.e., the Taurus dark cloud, reflection nebulae, etc., our results suggest that the alignment by RATs is ubiquitous in the interstellar space.
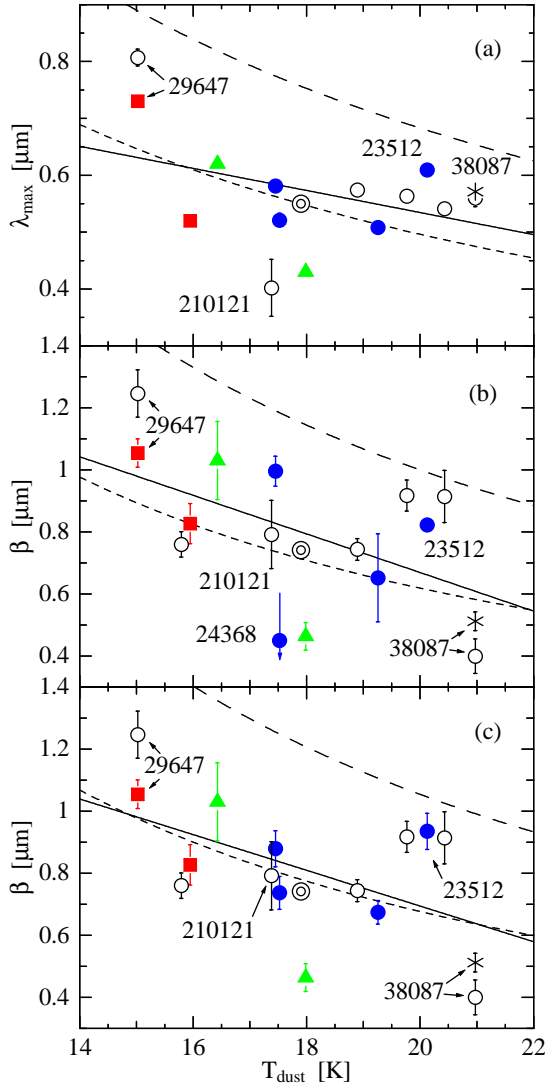
Fig. 3. (a) Correlation between T_{dust} and λ_{max}. Filled circles are the Pleiades stars, open circles the results by Weitenbeck (1999, 2004), filled squares those by Whittet et al. (2001), and filled triangles those by Seki & Matsumura (1996). Double circle is the average of interstellar medium. Result of linear fitting for 14 stars is drawn as solid line. Short dashed line presents a model prediction for $a' = a_{lower}$, while long dashed line $a' = 2a_{lower}$. See text for details. (b) Same as (a) but for T_{dust} and β with Method 1. (c) Same as (b) but with Method 2.

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